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由②$c\sin A=3$，所以$c=b=2\sqrt{3}$，$a=6$.

因此，选条件②时问题中的三角形存在，此时$c=2\sqrt{3}$.

方案三：选条件③.

由$C=\frac{\pi}{6}$和余弦定理得$\frac{a^2+b^2-c^2}{2ab}=\frac{\sqrt{3}}{2}$.

由$\sin A=\sqrt{3}\sin B$及正弦定理得$a=\sqrt{3}b$.

于是$\frac{3b^2+b^2-c^2}{2\sqrt{3}b^2}=\frac{\sqrt{3}}{2}$，由此可得$b=c$.

由③$c=\sqrt{3}b$，与$b=c$矛盾.

因此，选条件③时问题中的三角形不存在.

18. **解:**

（1）设$\{a_n\}$的公比为q．由题设得$a_1q+a_1q^3=20$，$a_1q^2=8$.

解得$q=\frac{1}{2}$（舍去），$q=2$．由题设得$a_1=2$.

所以$\{a_n\}$的通项公式为$a_n=2^n$.

（2）由题设及（1）知$b_1=0$，且当$2^n\leqslant m<2^{n+1}$时，$b_m=n$.

所以

$$S_{100}=b_1+(b_2+b_3)+(b_4+b_5+b_6+b_7)+\cdots+(b_{32}+b_{33}+\cdots+b_{63})+(b_{64}+b_{65}+\cdots+b_{100})$$
$$=0+1\times2+2\times2^2+3\times2^3+4\times2^4+5\times2^5+6\times(100-63)$$
$$=480.$$

19. **解:**

（1）根据抽查数据，该市100天的空气中PM2.5浓度不超过75，且SO_2浓度不超过150的天数为$32+18+6+8=64$，因此，该市一天空气中PM2.5浓度不超过75，且SO_2浓度不超过150的概率的估计值为$\frac{64}{100}=0.64$.

（2）根据抽查数据，可得2×2列联表：

PM2.5 \ SO_2	[0,150]	(150,475]
[0,75]	64	16
(75,115]	10	10

（3）根据（2）的列联表得

$$K^2=\frac{100\times(64\times10-16\times10)^2}{80\times20\times74\times26}\approx7.484.$$

由于$7.484>6.635$，故有99%的把握认为该市一天空气中PM2.5浓度与SO_2浓度有关.

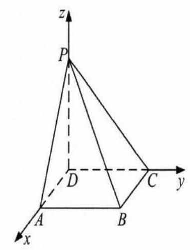
20. **解:**

（1）因为$PD\perp$底面$ABCD$，所以$PD\perp AD$．又底面$ABCD$为正方形，所以$AD\perp DC$．因此$AD\perp$平面PDC.

因为$AD//BC$，$AD\not\subset$平面PBC，所以$AD//$平面PBC．由已知得$l//AD$.

因此$l\perp$平面PDC.

（2）以D为坐标原点，$\overrightarrow{DA}$的方向为x轴正方向，建立如图所示的空间直角坐标系$D-xyz$．则$D(0,0,0)$，$C(0,1,0)$，$B(1,1,0)$，$P(0,0,1)$，$\overrightarrow{DC}=(0,1,0)$，$\overrightarrow{PB}=(1,1,-1)$.



由（1）可设$Q(a,0,1)$，则$\overrightarrow{DQ}=(a,0,1)$.

设$\boldsymbol{n}=(x,y,z)$是平面QCD的法向量，则

$$\begin{cases}\boldsymbol{n}\cdot\overrightarrow{DQ}=0,\\ \boldsymbol{n}\cdot\overrightarrow{DC}=0,\end{cases}\text{即}\begin{cases}ax+z=0,\\ y=0.\end{cases}$$

可取$\boldsymbol{n}=(-1,0,a)$.

所以$\cos\langle\boldsymbol{n},\overrightarrow{PB}\rangle=\frac{\boldsymbol{n}\cdot\overrightarrow{PB}}{|\boldsymbol{n}|\cdot|\overrightarrow{PB}|}=\frac{-1-a}{\sqrt{3}\sqrt{1+a^2}}$.

设PB与平面QCD所成角为θ，则$\sin\theta=\frac{\sqrt{3}}{3}\times\frac{|a+1|}{\sqrt{1+a^2}}=\frac{\sqrt{3}}{3}\sqrt{1+\frac{2a}{a^2+1}}$.

因为$\frac{\sqrt{3}}{3}\sqrt{1+\frac{2a}{a^2+1}}\leqslant\frac{\sqrt{6}}{3}$，当且仅当$a=1$时等号成立，所以$PB$与平面$QCD$所成角的正弦值的最大值为$\frac{\sqrt{6}}{3}$.

21. **解:**

$f(x)$的定义域为$(0,+\infty)$，$f'(x)=ae^{x-1}-\frac{1}{x}$.

（1）当$a=e$时，$f(x)=e^x-\ln x+1$，$f'(1)=e-1$，曲线$y=f(x)$在点$(1,f(1))$处的切线方程为$y-(e+1)=(e-1)(x-1)$，即 $y=(e-1)x+2$.

直线$y=(e-1)x+2$在x轴，y轴上的截距分别为$\frac{-2}{e-1}$，2.

因此所求三角形的面积为$\frac{2}{e-1}$.

（2）当$0<a<1$时，$f(1)=a+\ln a<1$.

当$a=1$时，$f(x)=e^{x-1}-\ln x$，$f'(x)=e^{x-1}-\frac{1}{x}$．当$x\in(0,1)$时，$f'(x)<0$；当$x\in(1,+\infty)$时，$f'(x)>0$．所以当$x=1$时，$f(x)$取得最小值，最小值为$f(1)=1$，从而$f(x)\geqslant1$.

当$a>1$时，$f(x)=ae^{x-1}-\ln x+\ln a\geqslant e^{x-1}-\ln x\geqslant1$.

综上，a的取值范围是$[1,+\infty)$.

22. **解:**

（1）由题设得$\frac{4}{a^2}+\frac{1}{b^2}=1$，$\frac{a^2-b^2}{a^2}=\frac{1}{2}$，解得$a^2=6$，$b^2=3$.

所以C的方程为$\frac{x^2}{6}+\frac{y^2}{3}=1$.

（2）设$M(x_1,y_1)$，$N(x_2,y_2)$.

若直线MN与x轴不垂直，设直线MN的方程为$y=kx+m$，代入$\frac{x^2}{6}+\frac{y^2}{3}=1$得

$$(1+2k^2)x^2+4kmx+2m^2-6=0.$$

于是

$$x_1+x_2=-\frac{4km}{1+2k^2},\quad x_1x_2=\frac{2m^2-6}{1+2k^2}.\qquad ①$$

由$AM\perp AN$知$\overrightarrow{AM}\cdot\overrightarrow{AN}=0$，故$(x_1-2)(x_2-2)+(y_1-1)(y_2-1)=0$，可得

$$(k^2+1)x_1x_2+(km-k-2)(x_1+x_2)+(m-1)^2+4=0.$$

将①代入上式可得$(k^2+1)\frac{2m^2-6}{1+2k^2}-(km-k-2)\frac{4km}{1+2k^2}+(m-1)^2+4=0$.

整理得$(2k+3m+1)(2k+m-1)=0$.

因为$A(2,1)$不在直线MN上，所以$2k+m-1\neq0$，故$2k+3m+1=0$，$k\neq1$.

于是MN的方程为$y=k(x-\frac{2}{3})-\frac{1}{3}\ (k\neq1)$.

所以直线MN过点$P(\frac{2}{3},-\frac{1}{3})$.

若直线MN与x轴垂直，可得$N(x_1,-y_1)$.

由$\overrightarrow{AM}\cdot\overrightarrow{AN}=0$得$(x_1-2)(x_1-2)+(y_1-1)(-y_1-1)=0$.

又$\frac{x_1^2}{6}+\frac{y_1^2}{3}=1$，可得$3x_1^2-8x_1+4=0$．解得$x_1=2$（舍去），$x_1=\frac{2}{3}$.

此时直线MN过点$P(\frac{2}{3},-\frac{1}{3})$.

令Q为AP的中点，即$Q(\frac{4}{3},\frac{1}{3})$.

若D与P不重合，则由题设知AP是$\mathrm{Rt}\triangle ADP$的斜边，故$|DQ|=\frac{1}{2}|AP|=\frac{2\sqrt{2}}{3}$.

若D与P重合，则$|DQ|=\frac{1}{2}|AP|$.

综上，存在点$Q(\frac{4}{3},\frac{1}{3})$，使得$|DQ|$为定值.

绝密★启用前

2020年普通高等学校招生全国统一考试

英　语

注意事项：

1. 答卷前，考生务必将自己的姓名、准考证号填写在答题卡上。
2. 回答选择题时，选出每小题答案后，用铅笔把答题卡上对应题目的答案标号涂黑。如需改动，用橡皮擦干净后，再选涂其他答案标号。回答非选择题时，将答案写在答题卡上，写在本试卷上无效。
3. 考试结束后，将本试卷和答题卡一并交回。

第一部分　阅读（共两节，满分50分）

第一节　（共15小题；每小题2.5分，满分37.5分）

阅读下列短文，从每题所给的A、B、C、D四个选项中选出最佳选项。

A

POETRY CHALLENGE

Write a poem about how courage, determination, and strength have helped you face challenges in your life.

Prizes

3 Grand Prizes: Trip to Washington, D.C. for each of three winners, a parent and one other person of the winner's choice. Trip includes round-trip air tickets, hotel stay for two nights, and tours of the National Air and Space Museum and the office of National Geographic World.

6 First Prizes: The book *Sky Pioneer: A Photobiography of Amelia Earhart* signed by author Corinne Szabo and pilot Linda Finch.

50 Honorable Mentions: Judges will choose up to 50 honorable mention winners, who will each receive a T-shirt in memory of Earhart's final flight.

Rules

Follow all rules carefully to prevent disqualification.

- Write a poem using 100 words or fewer. Your poem can be any format, any number of lines.

- Write by hand or type on a single sheet of paper. You may use both the front and back of the paper.
- On the same sheet of paper, write or type your name, address, telephone number, and birth date.
- Mail your entry to us by October 31 this year.

1. How many people can each grand prize winner take on the free trip?

A. Two.　　B. Three.　　C. Four.　　D. Six.

2. What will each of the honorable mention winners get?

A. A plane ticket.　　B. A book by Corinne Szabo.

C. A special T-shirt.　　D. A photo of Amelia Earhart.

3. Which of the following will result in disqualification?

A. Typing your poem out.　　B. Writing a poem of 120 words.

C. Using both sides of the paper.　　D. Mailing your entry on October 30.

B

Jennifer Mauer has needed more willpower than the typical college student to pursue her goal of earning a nursing degree. That willpower bore fruit when Jennifer graduated from University of Wisconsin-Eau Claire and became the first in her large family to earn a bachelor's degree.

Mauer, of Edgar, Wisconsin, grew up on a farm in a family of 10 children. Her dad worked at a job away from the farm, and her mother ran the farm with the kids. After high school, Jennifer attended a local technical college, working to pay her tuition (学费), because there was no extra money set aside for a college education. After graduation, she worked to help her sisters and brothers pay for their schooling.

Jennifer now is married and has three children of her own. She decided to go back to college to advance her career and to be able to better support her family while doing something she loves: nursing. She chose the UW-Eau Claire program at Ministry Saint Joseph's Hospital in Marshfield because she was able to pursue her four-year degree close to home. She could drive to class and be home in the evening to help with her kids. Jennifer received great support from her family as she worked to earn her degree: Her husband worked two jobs to cover the bills, and her 68-year-old mother helped take care of the children at times.